Digitization Error and Position Resolution

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The finite ADC bin width of the digitized pulse heights at each end of a CsI bar in the GLAST calorimeter will cause some position uncertainty. This uncertainty will clearly be negligible for large pulse heights, but near threshold or near the cross-over points in the various energy ranges it may be significant.

I have estimated the position uncertainty using two methods, first by propagation of errors under the assumption that digitization error is a statistical quantity, and second by examining worst-case binning scenarios.

For both methods I assume a linear position mapping, i.e. that the position x of the interaction in the CsI bar is given by

$$x = A\frac{L-R}{L+R}$$

where L and R are PIN diode pulse heights (in ADC bins) from the "left" and "right" ends of the bar, and A is a proportionality constant, the scale factor between ratio and position. For the sample of 8 bars crossing the center of the calorimeter array used in the SLAC 1997 beam test, the average scale factor is A = 67 cm.

1. Error propagation

Let's look at digitization error only in one end. If we assume the signals at the two ends are uncorrelated and that the rms binning error $\sigma_L = \sigma_R = 1/\sqrt{12}$, standard error propagation says

$$\sigma_x^2 = \left(\frac{\partial x}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial x}{\partial R}\right)^2 \sigma_R^2 = \left[\left(\frac{\partial x}{\partial L}\right)^2 + \left(\frac{\partial x}{\partial R}\right)^2\right]\sigma_L^2$$

where the partials are

$$\left|\frac{\partial x}{\partial L}\right| = A\left(\frac{1}{L+R} - \frac{L-R}{(L+R)^2}\right) \qquad \left|\frac{\partial x}{\partial R}\right| = A\left(\frac{1}{L+R} + \frac{L-R}{(L+R)^2}\right)$$

Now throughout the bar and especially near the center, $L \approx R$ and $\partial x/\partial L \approx \partial x/\partial R \approx A/(L+R) = 1/2L$, so that $\sigma_x \approx \sqrt{2}\, A\, \sigma_L / (L+R)$. We solve for the pulse heights L+R required to produce a given position error and find

$$L + R = \sqrt{2}A\frac{\sigma_L}{\sigma_x}$$

Requiring $\sigma_x = 0.1$ cm and taking A = 67 cm and $\sigma_L = 1/\sqrt{12}$, we find $(L+R) \approx 275$ bins, or in the middle of the bar $L \approx R \approx 140$ bins.

2. Worst-case binning

Let's pick a guess at the worst case binning error. I define w to be the bin width, w = 1. Assume that gains are perfectly balanced and that ADC bin edges are perfectly aligned. Then at a bin transition, the x and y ends will transition with opposite sense, e.g. the x end will increase by 1 bin while the y end will decrease by 1 bin. Thus

$$x = A\frac{(L\mp w)-(R\pm w)}{(L\mp w)+(R\pm w)} = x_0 \mp A\frac{2w}{L+R}$$

where P_0 is the position corresponding to the bin centers, and the position offset is therefore

$$\Delta x = A\frac{2w}{L+R}$$

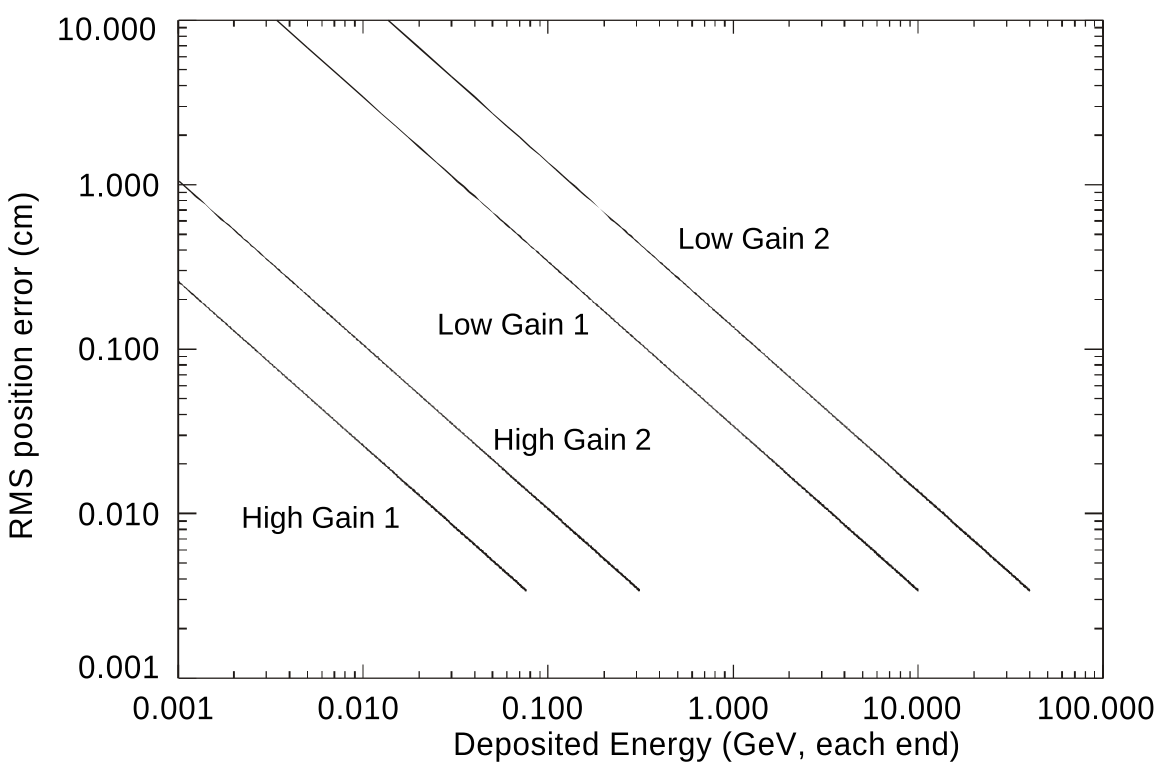
If we require a worst-case position offset of 0.3 cm and A = 67 cm, we find (L+R) ≥ 440 bins or in the middle of the bar that L = R ≥ 220 bins.

Conclusions

Both methods indicate that the position error induced by the finite ADC bin width will be a significant contributor below about 200 ADC bins. Alternatively we can express the rms position error as a function of ***deposited*** energy in a CsI bar using the current FEE and ADC design (I assume 12 useful bits) summarized in the following table.

	High Gain 1	**High Gain 2**	**Low Gain 1**	**Low Gain 2**
Energy Range	0 - 80 MeV	0 – 320 MeV	0 - 10.24 GeV	0 – 40.96 GeV
Channel Width	0.0019 MeV	0.078 MeV	2.5 MeV	10 MeV

I evaluated the expression for the rms position error and scaled ADC bin numbers by the above entries in the above table to give the deposited energy in each gain range. At the crossover point between the High Gain 2 and Low Gain 1 ranges, at 320 MeV deposited, the rms error is about 0.1 cm. At the other two crossover points, the rms error from digitization is smaller. At the low energy threshold, the rms error from digitization is presumably not the dominant source of position uncertainty.



Appendix: Pedestal Uncertainty and Differential Non-linearity

These expressions for position offsets can be used to estimate the error induced by uncertainty in the ADC pedestal value (or maybe more properly, the ADC bin that truly corresponds to zero input) or by differential non-linearity in the ADC.

From error propagation, an uncertainty in the pedestal at each end of $\sigma_L = 1/\sqrt{12}$ bins corresponds to a position uncertainty of $\sigma_x = \sqrt{2}\ A\ \sigma_L\ /\ (L+R) \approx 0.1$ cm at 140 bins, which is near the crossover between the

High Gain 2 and Low Gain 1 ranges. For larger pulse heights, the position error is smaller (it falls linearly with pulse height). Thus, assuming we can determine the pedestals to within $\sigma_L = 1/\sqrt{12}$ bins, this contribution to rms position error should always be less than 0.1 cm.

I expect that we will measure differential non-linearity for each of the 4000 ADCs in the calorimeter as part of the ground energy calibration of each channel. The residual uncertainty in the calibration of the channel edges should then presumably be less than $\sigma_L = 1/\sqrt{12}$ bins (rms), and therefore this contribution to rms position error should also always be less than 0.1 cm.